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Quantum and string shape fluctuations in the dual monopole Nambu–Jona–Lasinio model with dual Dirac strings

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Abstract. The magnetic monopole condensate is calculated in the dual Monopole Nambu–Jona–Lasinio model with dual Dirac strings suggested in [1,2] as a functional of the dual Dirac string shape. The calculation is carried out in the tree approximation in the scalar monopole–antimonopole collective excitation field. The integration over quantum fluctuations of the dual–vector monopole–antimonopole collective excitation field around the Abrikosov flux line and string shape fluctuations are performed explicitly. We claim that there are important contributions of quantum and string shape fluctuations to the magnetic monopole condensate.

1 Introduction

In [1,2] there has been suggested the dual Monopole Nambu–Jona–Lasinio (MNJL) model with dual Dirac strings as a continuum analogy of Compact Quantum Electrodynamics (CQED) which is defined for lattices as nonlinear $U(1)$ gauge theory. It has a confining phase like QCD [3] and realizes confinement of "color" electric charges. Thereby, the investigation of CQED should help us to understand quark confinement. As has been shown in [4] the non-perturbative vacuum of CQED behaves like an effective dual superconductor with magnetic monopoles. Due to magnetic monopoles the electric flux between quarks rearranges and looks like the field produced by a dual Dirac string. As a result quarks interact via a linearly rising potential [5,6] that realizes quark confinement [7,8] and spontaneous breaking of chiral symmetry (SBχS) [8].

The NJL model [9] can be regarded as some kind of relativistic extension of the BCS (Bardeen–Cooper–Schrieffer) theory of superconductivity [10]. It also possesses a non-perturbative vacuum with a ground state of the same kind as in a superconductor in the superconducting phase. The latter has been the promoting idea of [1,2] to put the NJL–model to the foundation of a continuum space–time model realizing non–perturbative phenomena of CQED.

The MNJL–model is based on the Lagrangian, invariant under "color" magnetic $U(1)$ symmetry, of massless magnetic monopoles, self–coupled through local four–monopole interaction [1,2]:

$$\mathcal{L}(x) = \bar{\chi}(x) i\gamma^{\mu}\partial_{\mu}\chi(x) + G[\bar{\chi}(x)\chi(x)]^2 - G_1[\bar{\chi}(x)\gamma_{\mu}\chi(x)][\bar{\chi}(x)\gamma^{\mu}\chi(x)], \tag{1.1}$$

where $\chi(x)$ is a massless magnetic "color" monopole field, G and G_1 are positive phenomenological constants responsible for the magnetic monopole condensation and the dual–"color" vector field mass, respectively.

The magnetic monopole condensation accompanies the creation of massive magnetic monopoles $\chi_M(x)$ with mass M, $\bar{\chi}\chi$–collective excitations with quantum numbers of scalar Higgs meson field σ with mass $M_{\sigma} = 2M$ and a massive dual–vector field C_{μ} with mass M_C defined as [1,2]:

$$M_C^2 = \frac{g^2}{2G_1} - \frac{g^2}{8\pi^2}[J_1(M) + M^2 J_2(M)], \tag{1.2}$$

where $J_1(M)$ and $J_2(M)$ are quadratically and logarithmically divergent integrals [1,2]

$$J_1(M) = \int \frac{d^4k}{\pi^2 i}\frac{1}{M^2 - k^2} = \Lambda^2 - M^2 \ell n\left(1 + \frac{\Lambda^2}{M^2}\right),$$
$$J_2(M) = \int \frac{d^4k}{\pi^2 i}\frac{1}{(M^2 - k^2)^2} = \ell n\left(1 + \frac{\Lambda^2}{M^2}\right) - \frac{\Lambda^2}{M^2 + \Lambda^2}. \tag{1.3}$$

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Here Λ is the ultra–violet cut–off. The mass of the magnetic monopole field $\chi_M(x)$ obeys the gap–equation [1,2]:

$$M = -2G < \bar{\chi}(0)\chi(0) >= \frac{GM}{2\pi^2} J_1(M). \quad (1.4)$$

After the integration over magnetic monopole degrees of freedom the effective Lagrangian containing quarks, antiquarks and the fields of scalar σ and dual–vector C_μ collective excitations reads

$$\begin{aligned}\mathcal{L}_{\rm eff}(x) &= \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \frac{1}{2}M_C^2 C_\mu(x)C^\mu(x)\\ &+\frac{1}{2}\partial_\mu\sigma(x)\partial^\mu\sigma(x) - \frac{1}{2}M_\sigma^2\sigma^2(x)\left[1+\kappa\frac{\sigma(x)}{M_\sigma}\right]^2\\ &+\mathcal{L}_{\rm free\ quark}(x). \end{aligned} \quad (1.5)$$

The gap–equation (1.4) has been derived from the effective Lagrangian (1.5) by virtue of the suppression of the direct transitions $\sigma \longleftrightarrow$ *vacuum* [1,2]. When matching the gap–equation $M_\chi = -2G < \bar{\chi}\chi >$ with that $M_\chi = -2(G_1 + 3G/4) < \bar{\chi}\chi >$ derived as one–loop corrections to the mass of the monopole field by using the Lagrangian (1.1) we fix G_1 in terms of G: $G_1 = G/4$.

The coupling constants g and κ are related by the constraint

$$\frac{g^2}{12\pi^2}J_2(M) = \frac{\kappa^2}{8\pi^2}J_2(M) = 1 \quad (1.6)$$

or $\kappa^2 = 2g^2/3$ [1,2]. $\mathcal{L}_{\rm free\ quark}(x)$ is the kinetic term for the quark and antiquark,

$$\begin{aligned}&\mathcal{L}_{\rm free\ quark}(x) =\\ &-\sum_{i=q,\bar{q}} m_i \int d\tau \left(\frac{dX_i^\mu(\tau)}{d\tau}\frac{dX_i^\nu(\tau)}{d\tau}g_{\mu\nu}\right)^{1/2}\\ &\delta^{(4)}(x - X_i(\tau))\end{aligned} \quad (1.7)$$

We consider quark and antiquark as classical point–like particles with masses $m_q = m_{\bar{q}} = m$, electric charges $Q_q = -Q_{\bar{q}} = Q$, and trajectories $X_q^\nu(\tau)$ and $X_{\bar{q}}^\nu(\tau)$, respectively. The field strength $F^{\mu\nu}(x)$ is defined [1,2] as $F^{\mu\nu}(x) = \mathcal{E}^{\mu\nu}(x) - {}^*dC^{\mu\nu}(x)$, where $dC^{\mu\nu}(x) = \partial^\mu C^\nu(x) - \partial^\nu C^\mu(x)$, and ${}^*dC^{\mu\nu}(x)$ is the dual version, i.e., ${}^*dC^{\mu\nu}(x) = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} dC_{\alpha\beta}(x)(\varepsilon^{0123} = 1)$.

The "color" electric field strength $\mathcal{E}^{\mu\nu}(x)$, induced by a dual Dirac string, is defined following [1,2] as

$$\begin{aligned}\mathcal{E}^{\mu\nu}(x) = Q\iint d\tau d\sigma \left(\frac{\partial X^\mu}{\partial\tau}\frac{\partial X^\nu}{\partial\sigma} - \frac{\partial X^\nu}{\partial\tau}\frac{\partial X^\mu}{\partial\sigma}\right)\\ \delta^{(4)}(x - X),\end{aligned} \quad (1.8)$$

where $X^\mu = X^\mu(\tau,\sigma)$ represents the position of a point on the world sheet swept by the string. The sheet is parameterized by the internal coordinates $-\infty < \tau < \infty$ and $0 \le \sigma \le \pi$, so that $X^\mu(\tau,0) = X^\mu_{-Q}(\tau)$ and $X^\mu(\tau,\pi) = X^\mu_Q(\tau)$ represent the world lines of an antiquark and a quark [1,2,5]. Within the definition (1.8) the tensor field $\mathcal{E}^{\mu\nu}(x)$ satisfies identically the equation of motion, $\partial_\mu F^{\mu\nu}(x) = J^\nu(x)$. The electric quark current $J^\nu(x)$ is defined as

$$J^\nu(x) = \sum_{i=q,\bar{q}} Q_i \int d\tau \frac{dX_i^\nu(\tau)}{d\tau}\delta^{(4)}(x - X_i(\tau)). \quad (1.9)$$

Hence, the inclusion of a dual Dirac string in terms of $\mathcal{E}^{\mu\nu}(x)$ defined by (1.8) satisfies completely the electric Gauss law of Dirac′s extension of Maxwell′s electrodynamics.

As has been shown in [1,2] the vacuum expectation values of time–ordered products of densities expressed in terms of the massless–monopole field, i.e., the magnetic monopole Green function

$$\begin{aligned}G\,(x_1,\ldots,x_n) = &< 0|{\rm T}(\bar{\chi}(x_1)\Gamma_1\chi(x_1)\\ &\ldots\bar{\chi}(x_n)\,\Gamma_n\chi\,(x_n))|0>_{\rm conn.}\end{aligned} \quad (1.10)$$

where $\Gamma_i(i = 1,\ldots,n)$ are the Dirac matrices, are given by [1,2]

$$\begin{aligned}G(x_1,\ldots,x_n) &= < 0|{\rm T}(\bar{\chi}(x_1)\Gamma_1\chi(x_1)\\ &\quad\ldots\bar{\chi}(x_n)\Gamma_n\chi(x_n))|0>_{\rm conn.}\\ &= {}^{(M)}< 0|{\rm T}\Big(\bar{\chi}_M(x_1)\Gamma_1\chi_M(x_1)\\ &\quad\ldots\bar{\chi}_M(x_n)\Gamma_n\chi_M(x_n)\times\ \exp i\int d^4x\\ &\quad -g\bar{\chi}_M(x)\gamma^\nu\chi_M(x)C_\nu(x)\\ &\quad -\kappa\bar{\chi}_M(x)\chi_M(x)\sigma(x)\\ &\quad +\mathcal{L}_{\rm int}[\sigma(x)]\}\Big)|0>^{(M)}_{\rm conn.}\ .\end{aligned} \quad (1.11)$$

Here $|0>^{(M)}$ is the wave-function of the non-perturbative vacuum of the MNJL–model in the condensed phase and $|0>$ the wave-function of the non-condensed perturbative vacuum. $\mathcal{L}_{\rm int}[\sigma(x)]$ describes self–interactions of the σ–field:

$$\mathcal{L}_{\rm int}[\sigma(x)] = -\kappa\,M_\sigma\,\sigma^3(x) - \frac{1}{2}\,\sigma^4(x). \quad (1.12)$$

The self–interactions $\mathcal{L}_{\rm int}[\sigma(x)]$ provide σ–field loop contributions and can be dropped out in the tree σ–field approximation accepted in [1,2]. In the tree σ–field approximation the r.h.s. of (1.11) acquires the form

$$\begin{aligned}G(x_1,\ldots,x_n) &= < 0|{\rm T}(\bar{\chi}(x_1)\Gamma_1\chi(x_1)\\ &\quad\ldots\bar{\chi}(x_n)\Gamma_n\chi(x_n))|0>_{\rm conn.}\\ &= {}^{(M)}< 0|{\rm T}\Big(\bar{\chi}_M(x_1)\Gamma_1\chi_M(x_1)\\ &\quad\ldots\bar{\chi}_M(x_n)\Gamma_n\chi_M(x_n)\\ &\quad\times\exp i\int d^4x\Big\{-g\bar{\chi}_M(x)\gamma^\nu\chi_M(x)C_\nu(x)\\ &\quad-\kappa\bar{\chi}_M(x)\chi_M(x)\sigma(x)\}\Big)|0>^{(M)}_{\rm conn.}\ .\end{aligned} \quad (1.13)$$

The tree σ–field approximation can be justified keeping massive magnetic monopoles very heavy, i.e. $M \gg M_C$.

This corresponds to the London limit $M_\sigma = 2\,M \gg M_C$ in the dual Higgs model with dual Dirac strings [11]. The inequality $M_\sigma \gg M_C$ means also that in the MNJL model we deal with *Dual Superconductivity of type* II [12].

In [2] (1.13) has been applied to the computation of the magnetic monopole condensate $<\bar\chi(x)\chi(x);\mathcal{E}>$ in dependence of the dual Dirac string shape represented by the electric string tensor $\mathcal{E}^{\mu\nu}(x)$ (1.8). The magnetic monopole condensate $<\bar\chi(x)\chi(x);\mathcal{E}>$ has been calculated in the tree σ–field approximation neglecting the fluctuations of the dual–vector field C_μ around the Abrikosov flux line which satisfies the equation

$$(\Box + M_C^2)C^\nu[\mathcal{E}(x)] = -\partial_\mu{}^*\mathcal{E}^{\mu\nu}(x), \qquad (1.14)$$

and takes the form

$$C^\nu[\mathcal{E}(x)] = -\int d^4x'\varDelta(x-x',M_C) \times\partial'_\mu{}^*\mathcal{E}^{\mu\nu}(x'), \qquad (1.15)$$

where $\varDelta(x-x',M_C)$ is the Green function

$$\varDelta(x-x',M_C) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot(x-x')}}{M_C^2-k^2-i0}. \qquad (1.16)$$

In this paper we calculate the magnetic monopole condensate $<\bar\chi(x)\chi(x);\mathcal{E}>$ in the tree σ–field approximation but taking into account quantum fluctuations of the dual–vector field C_μ around the Abrikosov flux line $C^\nu[\mathcal{E}(x)]$. An important role of such fluctuations for the formation of the interquark potential has been pointed out in [13] within a dual Higgs model with dual Dirac strings.

This paper is organized as follows: In Sect. 2 we calculate the magnetic monopole condensate in the tree σ–field approximation and explicitly integrate out quantum fluctuations of the dual–vector field C_μ around the Abrikosov flux line. In Sect. 3 we calculate the contribution of the string shape fluctuations to the magnetic monopole condensate. In the Conclusion we discuss the obtained results.

2 Quantum dual–vector field fluctuations

In the tree σ–field approximation we determine the magnetic monopole condensate $<\bar\chi(x)\chi(x);\mathcal{E}>$ following [1] as

$$\begin{aligned}&<\bar\chi(x)\chi(x);\mathcal{E}> - <\bar\chi(0)\chi(0)>\\ &={}^{(M)}\!\!< 0|\mathrm{T}\Big(\bar\chi_M(x)\chi_M(x)\\ &\times\ \exp i\int d^4z\Big\{-g\bar\chi_M(z)\gamma^\nu\chi_M(z)C_\nu(z)\\ &-\kappa\bar\chi_M(z)\chi_M(z)\sigma(z)\Big\}\Big)|0>^{(M)}_{\rm conn.},\end{aligned} \qquad (2.1)$$

where the r.h.s. of (2.1) should vanish at $C_\mu=\sigma=0$.

The time ordering operator and vacuum wave–function act on the massive magnetic monopole fields χ_M and the fields of collective excitations σ and C_μ.

The calculation of vacuum expectation values of time–ordered products of the dual–vector fields is convenient to perform by means of the path integral method

$$\begin{aligned}&<\bar\chi(x)\chi(x);\mathcal{E}> - <\bar\chi(0)\chi(0)>\\ &=\frac{1}{Z}\int\mathcal{D}C_\mu e^{i\int d^4z\,\mathcal{L}_{\rm eff}[C_\mu(z)]}\,{}^{(M)}\!\!< 0|\mathrm{T}\Big(\bar\chi_M(x)\chi_M(x)\\ &\times\exp i\int d^4z\Big\{-g\bar\chi_M(z)\gamma^\nu\chi_M(z)C_\nu(z)\\ &-\kappa\bar\chi_M(z)\chi_M(z)\sigma(z)\Big\}\Big)|0>^{(M)}_{\rm conn.},\end{aligned} \qquad (2.2)$$

where Z is a normalization factor determined as

$$Z = \int\mathcal{D}C_\mu e^{i\int d^4z\,\mathcal{L}_{\rm eff}[C_\mu(z)]}. \qquad (2.3)$$

The effective Lagrangian $\mathcal{L}_{\rm eff}[C_\mu(z)]$ is defined by the part of the effective Lagrangian equation (1.5) related to the C_μ–field:

$$\mathcal{L}_{\rm eff}[C_\mu(z)] = \frac{1}{4}F_{\mu\nu}(z)F^{\mu\nu}(z) +\frac{1}{2}M_C^2C_\mu(z)C^\mu(z). \qquad (2.4)$$

In order to integrate out quantum fluctuations of the dual–vector field C_μ around the shape of the Abrikosov flux line we split the C_μ–field into a classical field $C_\mu[\mathcal{E}(z)]$ induced by the Dirac string and quantum fluctuations $c_\mu(z)$ around that classical background. [13]:

$$C_\mu(z) = C_\mu[\mathcal{E}(z)] + c_\mu(z), \qquad (2.5)$$

where $C_\mu[\mathcal{E}(z)]$ satisfies (1.14), and $c_\mu(z)$ are the fluctuations of the dual–vector field having a vanishing vacuum expectation value $< c_\mu(z) >= 0$. Substituting the decomposition equation (2.5) in the Lagrangian equation (2.4) we arrive at the Lagrangian of the quantum fields $c_\mu(x)$ fluctuating around the Abrikosov flux line.

$$\mathcal{L}_{\rm eff}[C_\mu(z)] = \mathcal{L}_{\rm string}(z) + \frac{1}{2}\,c_\mu(z) \times\Big[(\Box + M_C^2)\,g^{\mu\nu} - \partial^\mu\partial^\nu\Big]\,c_\nu(z), \qquad (2.6)$$

where we have used (1.14). The Lagrangian of the dual Dirac string $\mathcal{L}_{\rm string}(z)$ is defined [5,11,13,14]

$$\int d^4z\mathcal{L}_{\rm string}(z) = \frac{1}{4}M_C^2\int\!\!\int d^4zd^4y\mathcal{E}_{\mu\alpha}(z) \times\varDelta^\alpha_\nu(z-y,M_C)\mathcal{E}^{\mu\nu}(y), \qquad (2.7)$$

where $\varDelta^\alpha_\nu(z-y,M_C) = (g^\alpha_\nu + 2\partial^\alpha\partial_\nu/M_C^2)\varDelta(z-y;M_C)$.

Since the Lagrangian (2.5) is Gaussian with respect to the c_μ–field, we are able to integrate out the c_μ–field exactly.

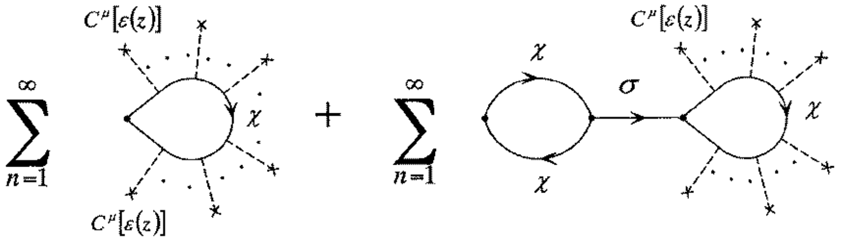

Fig. 1. Magnetic monopole diagrams describing the magnetic monopole condensate around the dual Dirac string without fluctuations of the dual–vector field

Integrating over the c_μ–field we reduce $< \bar\chi(x)\chi(x); \mathcal{E} >$ to the form:

$$
\begin{aligned}
&< \bar\chi(x)\chi(x); \mathcal{E} > - < \bar\chi(0)\chi(0) > =^{(M)}< 0|\mathrm{T} \\
&\times\Big(\bar\chi_M(x)\chi_M(x)\exp -i\,\frac{1}{2}\,g^2\iint d^4z d^4y\,[\bar\chi_M(z)\gamma^\mu\chi_M(z)] \\
&\times D_{\mu\nu}(z-z', M_C)\,[\bar\chi_M(z')\gamma^\nu\chi_M(z')] \\
&\times e^{\,i\int d^4z\{-g\bar\chi_M(z)\gamma^\nu\chi_M(z)C_\nu[\mathcal{E}(z)]} \\
&-\kappa\bar\chi_M(z)\chi_M(z)\sigma(z)\}\Big)|0>^{(M)}_{\mathrm{conn.}}, \qquad (2.8)
\end{aligned}
$$

where $D_{\mu\nu}(z-z', M_C)$ is the Green function of the free c_μ–field: $D_{\mu\nu}(z-z', M_C) = (g_{\mu\nu} + \partial_\mu\partial_\nu/M_C^2)\,\Delta(z-z', M_C)$. Since herein we consider dual Dirac strings as classical objects the contribution of the Lagrangian of the dual Dirac strings $\mathcal{L}_{\mathrm{string}}(z)$ cancels out.

The integration over σ–field degrees of freedom in the tree approximation we perform following [2]. This yields

$$
\begin{aligned}
&< \bar\chi(x)\chi(x); \mathcal{E} > - < \bar\chi(0)\chi(0) > \\
&= -\frac{\kappa^2}{4M^3}\ < \bar\chi(0)\chi(0) >\ ^{(M)}< 0|\mathrm{T}\Big(\bar\chi_M(x)\chi_M(x) \\
&\times\ \exp -i\frac{1}{2}g^2\iint d^4z d^4z'[\bar\chi_M(z)\gamma^\mu\chi_M(z)] \\
&D_{\mu\nu}(z-z'; M_C)\,[\bar\chi_M(z')\gamma^\nu\chi_M(z')] \qquad (2.9) \\
&\times\exp -i\,g\int d^4z\,[\bar\chi_M(z)\gamma^\nu\chi_M(z)]\,C_\nu[\mathcal{E}(z)]\Big)|0>^{(M)}_{\mathrm{conn.}},
\end{aligned}
$$

For the calculation of the vacuum expectation value in the r.h.s of (2.9) we assume that the massive magnetic monopole fields $\chi_M(x)$ are almost on–mass shell. It is valid due to a very large mass of the monopole fields. In this case the transfer momenta are small compared with the mass of the dual–vector field M_C. By virtue these assumptions we can reduce the four–monopole interaction in (2.9) to a point–like interaction.

$$
\begin{aligned}
&< \bar\chi(x)\chi(x); \mathcal{E} > - < \bar\chi(0)\chi(0) > \\
&= -\frac{\kappa^2}{4M^3}\ < \bar\chi(0)\chi(0) >\ ^{(M)}< 0|\mathrm{T}\Big(\bar\chi_M(x)\chi_M(x) \\
&\times\ \exp -i\int d^4z\Big\{\frac{g^2}{2M_C^2}[\bar\chi_M(z)\gamma^\mu\chi_M(z)]^2 \\
&+g\,[\bar\chi_M(z)\gamma^\nu\chi_M(z)]\,C_\nu[\mathcal{E}(z)]\Big\}\Big)|0>^{(M)}_{\mathrm{conn.}}\ . \qquad (2.10)
\end{aligned}
$$

Thus, since $M \gg M_C$ the vacuum averaging over the massive magnetic monopole fields can be represented by the momentum integrals [2] related to the magnetic monopole diagrams depicted in Fig. 1 and Fig. 2:

$$
\begin{aligned}
&^{(M)}< 0|\mathrm{T}\Big(\bar\chi_M(x)\chi_M(x)\times\exp -i\int d^4z\Big\{\frac{g^2}{2M_C^2}[\bar\chi_M(z)\gamma^\mu \\
&\times\chi_M(z)]^2 + g\,[\bar\chi_M(z)\gamma^\nu\chi_M(z)]\,C_\nu[\mathcal{E}(z)]\Big\}\Big)|0>^{(M)}_{\mathrm{conn.}} \\
&= -\frac{1}{16\pi^2}\int\frac{d^4k}{\pi^2 i}\mathrm{tr}\Big\{\frac{1}{M-\hat k+g\hat C[\mathcal{E}(x)]} - \frac{1}{M-\hat k}\Big\} \\
&-\frac{1}{16\pi^2}\int\frac{d^4k}{\pi^2 i}\mathrm{tr}\Big\{\frac{1}{M-\hat k+g\hat C[\mathcal{E}(x)]} \\
&\times\frac{1}{M-\hat k+g\hat C[\mathcal{E}(x)]}\gamma_{\mu_1}\Big\} \\
&\times\sum_{n=1}^{\infty}\left(\frac{g^2}{2M_C^2}\right)^n\left(\frac{1}{16\pi^2}\right)^{(n-1)}\int\frac{d^4k_1}{\pi^2 i}\mathrm{tr} \\
&\times\Big\{\frac{1}{M-\hat k_1+g\hat C[\mathcal{E}(x)]}\gamma_{\mu_1}\frac{1}{M-\hat k_1+g\hat C[\mathcal{E}(x)]}\gamma_{\mu_2}\Big\} \\
&\ldots\int\frac{d^4k_{n-1}}{\pi^2 i}\mathrm{tr}\Big\{\frac{1}{M-\hat k_{n-1}+g\hat C[\mathcal{E}(x)]}\gamma_{\mu_{n-1}} \\
&\times\frac{1}{M-\hat k_{n-1}+g\hat C[\mathcal{E}(x)]}\gamma_{\mu_n}\Big\}\frac{1}{16\pi^2}\int\frac{d^4k_n}{\pi^2 i}\mathrm{tr} \\
&\times\Big\{\gamma_{\mu_n}\frac{1}{M-\hat k_n+g\hat C[\mathcal{E}(x)]}\Big\} + \ldots . \qquad (2.11)
\end{aligned}
$$

The first term in the r.h.s. of (2.11) has been calculated in [2] at the neglect of quantum fluctuations of the dual–vector field C_μ, whereas the second term is fully due to these fluctuations. We calculate the second term keeping leading divergent contributions as it is accepted in the MNJL–model [1,2]. The ellipses denote the contribution of the diagrams depicted in Fig. 2b. This is a constant of order of $O(1/M)$ which can be removed by a slight redefinition of the cut–off and the magnetic monopole mass in order to retain the gap–equation (1.4) in the same form.

The vacuum expectation value (2.11) amounts to

$$
\begin{aligned}
&^{(M)}< 0|\mathrm{T}\Big(\bar\chi_M(x)\chi_M(x) \\
&\times\ \exp -i\int d^4z\Big\{\frac{g^2}{2M_C^2}[\bar\chi_M(z)\gamma^\mu\chi_M(z)]^2
\end{aligned}
$$

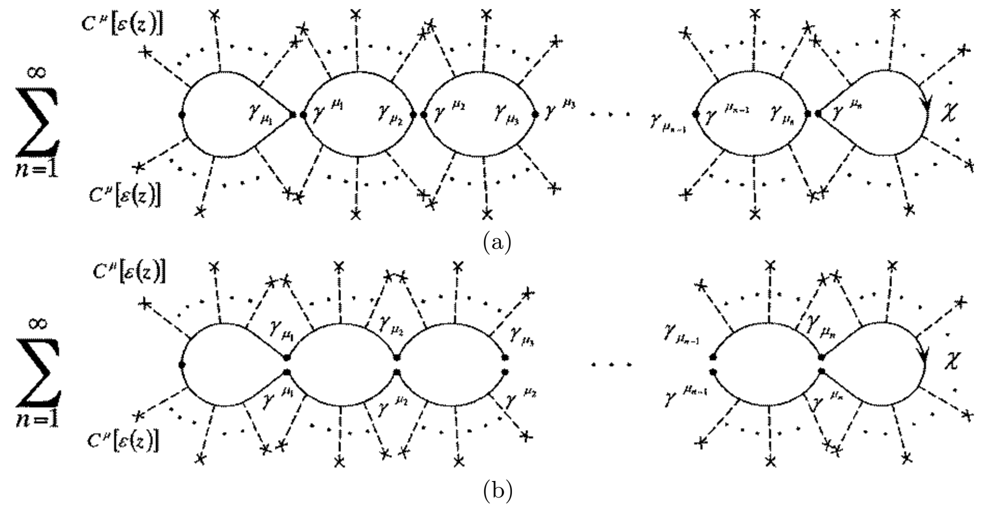

Fig. 2a, b. Magnetic monopole diagrams describing the contributions to the magnetic monopole condensate caused by the quantum fluctuations of the C_μ–field

$$+g\,[\bar{\chi}_M(z)\gamma^\nu\chi_M(z)]\,C_\nu[\mathcal{E}(z)]\Big\}\Big)|0>^{(M)}_{\rm conn.}$$
$$= -\frac{M}{8\pi^2}\,g^2\,C_\mu[\mathcal{E}(x)]C^\mu[\mathcal{E}(x)]$$
$$+\frac{M}{4\pi^2}\Big\{-\frac{g^2}{8\pi^2}[J_1(M)+M^2J_2(M)]\Big\}$$
$$\times\,\frac{g^2}{2M^2_C}\sum^\infty_{n=1}\Big\{\frac{1}{M^2_C}\frac{g^2}{16\pi^2}[J_1(M)+M^2J_2(M)]\Big\}^{(n-1)}$$
$$\times C_\mu[\mathcal{E}(x)]C^\mu[\mathcal{E}(x)]$$
$$= -\frac{M}{24\pi^2}\,\frac{M^2_C+\dfrac{g^2}{2G_1}}{M^2_C-\dfrac{g^2}{6G_1}}\,g^2\,C_\mu[\mathcal{E}(x)]C^\mu[\mathcal{E}(x)]. \qquad (2.12)$$

Here we have used the definitions of M^2_C given by (1.2).

Thus, integrating out explicitly quantum fluctuations of the dual–vector field C_μ around the Abrikosov flux line and taking into account the contribution of the scalar field σ in the tree approximation we obtain the magnetic monopole condensate in dependence on the shape of a dual Dirac string in the form:

$$<\bar{\chi}(x)\chi(x);\mathcal{E}> = <\bar{\chi}(0)\chi(0)>\Bigg\{1+\frac{M^2_C+\dfrac{g^2}{2G_1}}{M^2_C-\dfrac{g^2}{6G_1}}$$
$$\times\frac{\kappa^2}{96\pi^2}\,\frac{1}{M^2}\,g^2\,C_\mu[\mathcal{E}(x)]C^\mu[\mathcal{E}(x)]\Bigg\}. \qquad (2.13)$$

The non–trivial term in the braces of (2.13) is the result of the calculation of the diagrams in Fig. 2b. It may be seen that quantum fluctuations of a dual–vector field around the Abrikosov flux line give a substantial contribution to the magnetic monopole condensate. In order to retain the agreement with the results obtained within CQED [15] which testify the suppression of the magnetic monopole condensate in the region close to a dual Dirac string we have to impose the constraint $M^2_C > g^2/6G_1$.

3 Dual Dirac string shape fluctuations

The string shape fluctuations we define following [14,16] by $X^\mu \to X^\mu + \eta^\mu(X)$, where $\eta^\mu(X)$ describes fluctuations around the fixed surface S swept by the shape Γ and obeys the constraint $\eta^\mu(X)|_{\partial S} = 0$ [14,16] at the boundary ∂S of the surface S.

The magnetic monopole condensate defined by (2.13) and averaged over string shape fluctuations reads

$$<\bar{\chi}(x)\chi(x);\mathcal{E}> - <\bar{\chi}(0)\chi(0)>$$
$$= <\bar{\chi}(0)\chi(0)>\,\frac{M^2_C+\dfrac{g^2}{2G_1}}{M^2_C-\dfrac{g^2}{6G_1}}\,\frac{\kappa^2}{96\pi^2}\,\frac{1}{M^2}\,\frac{1}{Z_{\rm shape}}$$
$$\times\int\mathcal{D}\eta_\mu\,e^{i\,\delta\mathcal{S}_{\rm N}[\eta]}\,g^2\,C_\nu\{\eta(x)\}C^\nu\{\eta(x)\}, \qquad (3.1)$$

where $Z_{\rm shape}$ is a normalization factor determined as

$$Z_{\rm shape} = \int\mathcal{D}\eta_\mu e^{i\,\delta\mathcal{S}_{\rm N}[\eta]} \qquad (3.2)$$

and $\delta\mathcal{S}_{\rm N}[\eta]$ has been calculated in [14]:

$$\delta\,\mathcal{S}_{\rm N}[\eta] = \iint d^4x\,d^4y\,\eta_\alpha(x)\,O^{\alpha\beta}(x-y)\,\eta_\beta(y)$$
$$+\int d^4x\,\eta_\alpha(x)\,O^\alpha(x). \qquad (3.3)$$

The operators $O^{\alpha\beta}(x-y)$ and $O^\alpha(x)$ are given by

$$O^{\alpha\beta}(x-y) = \frac{1}{2}\,\delta^{(4)}(x-y)\,\mathcal{E}_{\mu\nu}(x)\,\frac{\partial^2}{\partial x_\alpha\partial x_\beta}\,\Sigma^{\nu\mu}(x)$$
$$+\frac{1}{4}\,M^2_C\,\mathcal{E}_{\mu\nu}(x)\,\frac{\partial^2}{\partial x_\alpha\partial x_\beta}\,\Delta^\nu{}_\lambda(x-y;M_C)\,\mathcal{E}^{\lambda\mu}(y)$$

$$
\begin{aligned}
&-\frac{\overleftarrow{\partial}}{\partial x_\mu}\,\delta^{(4)}(x-y)\,\mathcal{E}_{\mu\nu}(x)\,\Sigma^{\alpha\beta}(x)\,\frac{\overrightarrow{\partial}}{\partial y_\nu}\\
&+\frac{\overleftarrow{\partial}}{\partial x_\mu}\,\mathcal{E}_{\mu\nu}(x)\,\frac{\partial}{\partial x_\beta}\Sigma^{\nu\alpha}(x)\,\delta^{(4)}(x-y)\\
&-\frac{1}{2}\,M_C^2\,\frac{\overleftarrow{\partial}}{\partial x_\mu}\,\mathcal{E}_{\mu\nu}(x)\,\frac{\partial}{\partial x_\beta}\Delta^{\alpha}{}_{\lambda}(x-y;M_C)\,\mathcal{E}^{\lambda\nu}(y)\\
&+\delta^{(4)}(x-y)\,\frac{\partial}{\partial x_\alpha}\Sigma^{\beta\mu}(x)\,\mathcal{E}_{\mu\nu}(x)\,\frac{\overrightarrow{\partial}}{\partial y_\nu}\\
&+\frac{1}{2}\,M_C^2\,\frac{\overleftarrow{\partial}}{\partial x_\mu}\mathcal{E}_{\mu\nu}(x)\,\frac{\partial}{\partial x_\beta}\Delta^{\nu}{}_{\lambda}(x-y;M_C)\,\mathcal{E}^{\lambda\alpha}(y)\\
&-\frac{1}{4}\,M_C^2\,\frac{\overleftarrow{\partial}}{\partial x_\mu}\,\mathcal{E}_{\mu\lambda}(x)\,\Delta^{\alpha\beta}(x-y;M_C)\,\mathcal{E}^{\lambda\nu}(y)\,\frac{\overrightarrow{\partial}}{\partial y^\nu}\\
&-\frac{1}{4}\,M_C^2\,\frac{\overleftarrow{\partial}}{\partial x^\mu}\,g^{\alpha\beta}\,\mathcal{E}^{\mu\lambda}(x)\,\Delta_{\lambda\rho}(x-y;M_C)\,\mathcal{E}^{\rho\nu}(y)\,\frac{\overrightarrow{\partial}}{\partial y^\nu}\\
&+\frac{1}{4}\,M_C^2\,\frac{\overleftarrow{\partial}}{\partial x_\mu}\,\mathcal{E}^{\mu\lambda}(x)\,\Delta_{\lambda\beta}(x-y;M_C)\,\mathcal{E}^{\alpha\nu}(y)\,\frac{\overrightarrow{\partial}}{\partial y^\nu}\\
&+\frac{1}{4}\,M_C^2\,\frac{\overleftarrow{\partial}}{\partial x^\mu}\,\mathcal{E}^{\mu\beta}(x)\,\Delta^{\alpha}{}_{\lambda}(x-y;M_C)\,\mathcal{E}^{\lambda\nu}(y)\,\frac{\overrightarrow{\partial}}{\partial y^\nu},\\
&O^\alpha(x)=\mathcal{E}_{\mu\nu}(x)\frac{\partial}{\partial x_\alpha}\Sigma^{\nu\mu}(x)+\frac{1}{2}\,\mathcal{E}_{\mu\nu}(x)\frac{\partial}{\partial x_\mu}\Sigma^{\nu\alpha}(x)\\
&-\frac{1}{2}\,\mathcal{E}_{\mu\nu}(x)\,\frac{\partial}{\partial x_\mu}\Sigma^{\alpha\nu}(x),
\end{aligned}
\tag{3.4}
$$

where $\Sigma^{\nu\mu}(x)$ is determined by

$$
\Sigma^{\nu\mu}(x)=\frac{1}{2}\,M_C^2\int d^4z\,\Delta^{\mu}{}_{\lambda}(x-z;M_C)\,\mathcal{E}^{\lambda\nu}(z). \tag{3.5}
$$

Using (1.8) and (1.15) we determine $g^2\,C_\mu\{\eta(x)\}C^\mu\{\eta(x)\}$ as follows:

$$
\begin{aligned}
&g^2\,C_\mu\{\eta(x)\}C^\mu\{\eta(x)\}\\
&=g^2Q^2\iint d^*\sigma^{\lambda\mu}(X)d^*\sigma_{\rho\mu}(Y)\frac{\partial}{\partial x^\lambda}\\
&\quad\times\Delta(x-X-\eta(X))\frac{\partial}{\partial x_\rho}\Delta(x-Y-\eta(Y)).
\end{aligned}
\tag{3.6}
$$

The changes of the surface elements $\sigma^{\lambda\mu}(X)$ and $d\sigma_{\rho\mu}(Y)$ caused by the shifts $X\to X+\eta(X)$ and $Y\to Y+\eta(Y)$ have not been taken into account in the r.h.s. of (3.6), since they vanish for the straight string. Indeed, the integration over the η–field we perform following [14,16] for fluctuations around the shape of the static straight string with the length L tracing out the rectangular surface S with the time–side T. In this case the electric field strength $\mathcal{E}_{\mu\nu}(x)$ does not depend on time and reads

$$
\vec{\mathcal{E}}(\vec{x})=\mathbf{e}_z\,Q\,\delta(x)\,\delta(y)\times\left[\theta\left(z-\frac{1}{2}\,L\right)-\theta\left(z+\frac{1}{2}\,L\right)\right], \tag{3.7}
$$

where at $\mathbf{X}_q=(0,0,\frac{1}{2}\,L)$ and $\mathbf{X}_{\bar q}=(0,0,-\frac{1}{2}\,L)$ quark and antiquark are placed, respectively. Then the unit vector $\mathbf{e}_z$ is directed along the z–axis and $\theta(z)$ is the Heaviside–step–function. The field strength (3.7) induces the dual–vector potential

$$
<\mathbf{C}(\mathbf{x})>=-\,i\,Q\int\frac{d^3k}{4\,\pi^3}\,\frac{\mathbf{k}\times\mathbf{e}_z}{k_z}\times\frac{1}{M_C^2+\mathbf{k}^2}\,\sin\left(\frac{k_zL}{2}\right)e^{i\,\mathbf{k}\cdot\mathbf{x}}. \tag{3.8}
$$

Allowing only fluctuations in the plane perpendicular to the string world–sheet, i.e. setting $\eta_t(t,z)=\eta_z(t,z)=0$ [14,16], we arrive at the fluctuation action $\delta\,\mathcal{S}_{\rm N}[\eta_x,\eta_y]$

$$
\begin{aligned}
&\delta\,\mathcal{S}_{\rm N}[\eta_x,\eta_y]\\
&=\int_{-T/2}^{T/2}dt\int_{-T/2}^{T/2}dt'\int_{-L/2}^{L/2}dz\int_{-L/2}^{L/2}dz'\\
&\times\Bigg[\frac{\partial\eta_x(t,z)}{\partial t}\,O_1(t,z|t',z')\,\frac{\partial\eta_x(t',z')}{\partial t'}\\
&-\frac{\partial\eta_x(t,z)}{\partial z}\,O_2(t,z|t',z')\,\frac{\partial\eta_x(t',z')}{\partial z'}\\
&+\eta_x(t,z)\,O_3(t,z|t',z')\,\eta_x(t',z')+(x\leftrightarrow y)\Bigg],
\end{aligned}
\tag{3.9}
$$

where the operators O_i $(i=1,2,3)$ are defined by

$$
\begin{aligned}
&O_1(t,z|t',z')=Q^2\int\frac{d^2k_\perp}{64\pi^4}\\
&\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dk_0dk_z\,e^{-ik_0(t-t')+ik_z(z-z')}\\
&\times\frac{M_C^2+k_z^2+\frac{1}{2}\,\mathbf{k}_\perp^2}{M_C^2-k_0^2+k_z^2+\mathbf{k}_\perp^2},\\
&O_2(t,z|t',z')=Q^2\int\frac{d^2k_\perp}{64\pi^4}\\
&\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dk_0dk_z\,e^{-ik_0(t-t')+ik_z(z-z')}\\
&\times\frac{M_C^2-k_0^2+\frac{1}{2}\,\mathbf{k}_\perp^2}{M_C^2-k_0^2+k_z^2+\mathbf{k}_\perp^2},\\
&O_3(t,z|t',z')=\delta(t-t')\,\delta(z-z')\,Q^2\int\frac{d^2k_\perp\mathbf{k}_\perp^2}{16\pi^3}\\
&\times\int_{-\infty}^{\infty}\frac{dk_z}{k_z}\,\sin\left(\frac{k_zL}{2}\right)\cos(k_zz)\\
&\times\frac{M_C^2+k_z^2}{M_C^2+\mathbf{k}_\perp^2+k_z^2}-Q^2\int\frac{d^2k_\perp\mathbf{k}_\perp^2}{64\pi^4}\\
&\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dk_0dk_z\,e^{-ik_0(t-t')+ik_z(z-z')}
\end{aligned}
$$

$$\frac{M_C^2 - k_0^2 + k_z^2}{M_C^2 - k_0^2 + \mathbf{k}_\perp^2 + k_z^2}. \tag{3.10}$$

The linear terms in the η–field expansion do not appear, since only the components $\Sigma_{tz}(x)$ and $\Sigma_{zt}(x)$ survive in (3.4) for the static string strained along the z–axis.

The fluctuating fields $\eta_i(t,z)$, where $i = x, y$, should obey the boundary conditions $\eta_i(t,z)|_{\partial S} = 0$, which for the rectangular surface read [14,16]

$$\begin{aligned}\eta_i(t,z)|_{\partial S} &= \eta_i(\pm T/2, z) = \eta_i(t, \pm L/2)\\ &= \eta_i(\pm T/2, \pm L/2) = 0.\end{aligned} \tag{3.11}$$

The integration over the η–fields should be performed with the weight

$$\frac{1}{Z_{\rm shape}} \iint \mathcal{D}\eta_x \mathcal{D}\eta_y \, e^{i\delta\,\mathcal{S}_{\rm N}[\eta_x,\eta_y]}, \tag{3.12}$$

where the measure of the integration reads

$$\begin{aligned}\mathcal{D}\eta_x\mathcal{D}\eta_y = &\prod_{-T/2\le t\le T/2}\ \prod_{-L/2\le z\le L/2}\\ &\times d\eta_x(t,z)\, d\eta_y(t,z).\end{aligned} \tag{3.13}$$

Before the integration over the η–fields we can make some simplifications of the Δ–functions. For this aim we suggest to integrate out $\mathbf{k}_\perp$ keeping only the main divergent contributions as it is accepted in our effective approach [1,2]. In the region $-L/2 \le z \le L/2$ this reduces the operators O_i $(i = 1,2,3)$ to the expressions

$$\begin{aligned}O_1(t,z|t',z') &= O_2(t,z|t',z')\\ &= \frac{Q^2\Lambda_\perp^2}{32\pi}\,\delta(t-t')\,\delta(z-z'),\\ O_3(t,z|t',z') &= \frac{Q^2\Lambda_\perp^2}{16\pi}\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2}\right)\\ &\quad\times\delta(t-t')\,\delta(z-z'),\end{aligned} \tag{3.14}$$

where $\Lambda_\perp$ is the cut–off in the plane perpendicular to the world–sheet of the string. The fluctuation action becomes

$$\begin{aligned}\delta\,\mathcal{S}_{\rm N}[\eta_x,\eta_y] = &-\frac{3Q^2\Lambda_\perp^2}{32\pi}\int\limits_{-T/2}^{T/2} dt \int\limits_{-L/2}^{L/2} dz[\eta_x(t,z)\\ &\times(-\Delta)\,\eta_x(t,z) + (x \leftrightarrow y)],\end{aligned} \tag{3.15}$$

where Δ is the Laplace operator in 2–dimensional space–time

$$\Delta = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2} \tag{3.16}$$

The common factor $Q^2\Lambda_\perp^2/8\pi$ can be removed by the renormalization of the η–fields, and the action of the fluctuations becomes

$$\begin{aligned}\delta\,\mathcal{S}_{\rm N}[\eta_x,\eta_y] = &-\int\limits_{-T/2}^{T/2} dt \int\limits_{-L/2}^{L/2} dz\\ &\times\left[\eta_x(t,z)\left(-\frac{\Delta}{M_C^2}\right)\eta_x(t,z) + (x \leftrightarrow y)\right].\end{aligned} \tag{3.17}$$

The factor $1/M_C^2$ is introduced by dimensional considerations. We have used the mass of the dual–vector field, since the Abrikosov flux line is localized in the region of order of $O(1/M_C)$ in the xy–plane. Of course, the final result does not depend on the parameter making the operator Δ dimensionless.

For a static dual Dirac string and after the renormalization of the η–fields the scalar product $g^2\,C_\mu\{\eta(x)\}C^\mu\{\eta(x)\}$ amounts to

$$\begin{aligned}g^2\,C_\mu\{\eta(x)\}C^\mu\{\eta(x)\} &= g^2Q^2\\ &\times\iint\frac{d^3k}{4\pi^3}\frac{d^3q}{4\pi^3}\frac{k_xq_x + k_yq_y}{k_zq_z}\\ &\times\sin\left(\frac{k_zL}{2}\right)\sin\left(\frac{q_zL}{2}\right)\frac{1}{M_C^2+\mathbf{k}^2}\\ &\times\frac{1}{M_C^2+\mathbf{q}^2}\,e^{i\,(\mathbf{k}+\mathbf{q})\cdot\mathbf{x}}\,\exp i\sqrt{\frac{8\pi}{3}}\frac{1}{Q}\frac{2}{M_C\Lambda_\perp}\\ &\times[(k_x+q_x)\eta_x(t,z) + (k_y+q_y)\eta_y(t,z)].\end{aligned} \tag{3.18}$$

Thus, in the static dual Dirac string approximation equation (3.1) reads

$$\begin{aligned}&<\bar\chi(x)\chi(x);\mathcal{E}> - <\bar\chi(0)\chi(0)>\\ &=<\bar\chi(0)\chi(0)>\frac{M_C^2+\dfrac{g^2}{2G_1}}{M_C^2-\dfrac{g^2}{6G_1}}\frac{\kappa^2}{96\pi^2}\frac{1}{M^2}\\ &\times g^2Q^2\iint\frac{d^3k}{4\pi^3}\frac{d^3q}{4\pi^3}\frac{k_xq_x+k_yq_y}{k_zq_z}\sin\left(\frac{k_zL}{2}\right)\\ &\times\sin\left(\frac{q_zL}{2}\right)\frac{1}{M_C^2+\mathbf{k}^2}\frac{1}{M_C^2+\mathbf{q}^2}\,e^{i\,(\mathbf{k}+\mathbf{q})\cdot\mathbf{x}}\\ &\times\frac{1}{Z_{\rm shape}}\int\mathcal{D}\eta_x\mathcal{D}\eta_y\,\exp -i\int\limits_{-T/2}^{T/2}dt'\int\limits_{-L/2}^{L/2}dz'\\ &\times\Bigg[\eta_x(t',z')\left(-\frac{\Delta}{M_C^2}\right)\eta_x(t',z')\\ &-\sqrt{\frac{8\pi}{3}}\frac{1}{Q}\frac{2}{M_C\Lambda_\perp}(k_x+q_x)\delta(t-t')\\ &\times\delta(z-z')\,\eta_x(t',z') + (x\leftrightarrow y)\Bigg].\end{aligned} \tag{3.19}$$

Integrating over the η–fields we get

$$\begin{aligned}&<\bar\chi(x)\chi(x);\mathcal{E}> - <\bar\chi(0)\chi(0)>\\ &=<\bar\chi(0)\chi(0)>\frac{M_C^2+\dfrac{g^2}{2G_1}}{M_C^2-\dfrac{g^2}{6G_1}}\frac{\kappa^2}{96\pi^2}\frac{1}{M^2}\\ &\times g^2Q^2\iint\frac{d^3k}{4\pi^3}\frac{d^3q}{4\pi^3}\frac{\mathbf{k}_\perp\cdot\mathbf{q}_\perp}{k_zq_z}\sin\left(\frac{k_zL}{2}\right)\end{aligned}$$

$$\times \sin\left(\frac{q_z L}{2}\right)\frac{1}{M_C^2+\mathbf{k}^2}\,\frac{1}{M_C^2+\mathbf{q}^2}\,e^{i\,(\mathbf{k}+\mathbf{q})\cdot\mathbf{x}}$$

$$\times \exp\left\{-i\frac{8\pi}{3}\frac{(\mathbf{k}_\perp+\mathbf{q}_\perp)^2}{Q^2\Lambda_\perp^2}\int\limits_{-\infty}^{\infty}dt'\int\limits_{-L/2}^{L/2}dz'\delta(t-t')\right.$$

$$\left.\times\delta(z-z')\Delta^{-1}\delta(t-t')\delta(z-z')\right\}, \qquad (3.20)$$

where $\mathbf{k}_\perp\cdot\mathbf{q}_\perp = k_x q_x + k_y q_y$.

In the integrand the Green function $\Delta^{-1}\delta(t-t')\,\delta(z-z')$ should be calculated at certain boundary conditions. For the open dual Dirac string the calculations should be performed using Dirichlet boundary conditions [13,14]. Since in this case $\delta(z-z')$ is given by

$$\delta(z-z') = \frac{2}{L}\sum_{n=-\infty}^{\infty}$$

$$\times\sin\left(\frac{2\pi n}{L}z\right)\sin\left(\frac{2\pi n}{L}z'\right), \qquad (3.21)$$

the Green function $\Delta^{-1}\delta(t-t')\,\delta(z-z')$ is defined

$$\Delta^{-1}\delta(t-t')\,\delta(z-z')$$

$$= \frac{2}{L}\sum_{n=-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{e^{-i\omega(t-t')}}{\omega^2-\dfrac{4\pi^2n^2}{L^2}}$$

$$\times\sin\left(\frac{2\pi n}{L}z\right)\sin\left(\frac{2\pi n}{L}z'\right). \qquad (3.22)$$

Using (3.22) we reduce (3.20) to the expression

$$< \bar{\chi}(x)\chi(x);\mathcal{E} > - < \bar{\chi}(0)\chi(0) >$$

$$=< \bar{\chi}(0)\chi(0) > \frac{M_C^2+\dfrac{g^2}{2G_1}}{M_C^2-\dfrac{g^2}{6G_1}}\frac{\kappa^2}{96\pi^2}\frac{1}{M^2}$$

$$\times g^2Q^2\iint\frac{d^3k}{4\pi^3}\frac{d^3q}{4\pi^3}\frac{\mathbf{k}_\perp\cdot\mathbf{q}_\perp}{k_zq_z}$$

$$\times\sin\left(\frac{k_zL}{2}\right)\sin\left(\frac{q_zL}{2}\right)\frac{1}{M_C^2+\mathbf{k}^2}\,\frac{1}{M_C^2+\mathbf{q}^2}$$

$$\times e^{i\,(\mathbf{k}+\mathbf{q})\cdot\mathbf{x}}\exp\left\{-i\frac{8\pi}{3}\frac{(\mathbf{k}_\perp+\mathbf{q}_\perp)^2}{Q^2\Lambda_\perp^2}\right. \qquad (3.23)$$

$$\left.\times\frac{2}{L}\sum_{n=-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{\omega^2-\dfrac{4\pi^2n^2}{L^2}}\sin^2\left(\frac{2\pi n}{L}z\right)\right\}.$$

By applying the Wick rotation $\omega\to i\omega$ we obtain the magnetic monopole condensate in the form

$$< \bar{\chi}(x)\chi(x);\mathcal{E} > - < \bar{\chi}(0)\chi(0) >$$

$$=< \bar{\chi}(0)\chi(0) > \frac{M_C^2+\dfrac{g^2}{2G_1}}{M_C^2-\dfrac{g^2}{6G_1}}\frac{\kappa^2}{96\pi^2}\frac{1}{M^2}$$

$$\times\iint\frac{d^3k}{2\pi^2}\frac{d^3q}{2\pi^2}\frac{\mathbf{k}_\perp\cdot\mathbf{q}_\perp}{k_zq_z}\sin\left(\frac{k_zL}{2}\right)$$

$$\times\sin\left(\frac{q_zL}{2}\right)\frac{1}{M_C^2+\mathbf{k}^2}\,\frac{1}{M_C^2+\mathbf{q}^2}\,e^{i\,(\mathbf{k}+\mathbf{q})\cdot\mathbf{x}}$$

$$\times\exp\left\{-\frac{1}{2}\frac{(\mathbf{k}_\perp+\mathbf{q}_\perp)^2}{\Lambda_\perp^2}\varphi(z)\right\}, \qquad (3.24)$$

where we have used the Dirac quantization condition $g\,Q = 2\pi$ and denoted

$$\varphi(z) = \frac{4}{3}\frac{g^2}{\pi}\frac{2}{L}\sum_{n=-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\frac{d\omega}{2\pi}$$

$$\times\frac{1}{\omega^2+\dfrac{4\pi^2n^2}{L^2}}\sin^2\left(\frac{2\pi n}{L}z\right). \qquad (3.25)$$

The function $\varphi(z)$ is defined by a divergent series. Therefore, it should be regularized. The regularization of this function we perform in the Appendix. As it is shown the regularized $\varphi(z)$–function equals to zero for any z ranging the values from the interval $-L/2 \le zL/2$. Thus, below we set $\varphi(z)=0$ and get

$$< \bar{\chi}(x)\chi(x);\mathcal{E} > - < \bar{\chi}(0)\chi(0) >$$

$$=< \bar{\chi}(0)\chi(0) > \frac{M_C^2+\dfrac{g^2}{2G_1}}{M_C^2-\dfrac{g^2}{6G_1}}\frac{\kappa^2}{96\pi^2}\frac{1}{M^2}$$

$$\times\iint\frac{d^3k}{2\pi^2}\frac{d^3q}{2\pi^2}\frac{\mathbf{k}_\perp\cdot\mathbf{q}_\perp}{k_zq_z}\sin\left(\frac{k_zL}{2}\right)\sin\left(\frac{q_zL}{2}\right)$$

$$\times\frac{1}{M_C^2+\mathbf{k}^2}\,\frac{1}{M_C^2+\mathbf{q}^2}\,e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{x}}. \qquad (3.26)$$

For a sufficiently long string the main contributions to the integrals over k_z and q_z come from the momenta $|k_z|\sim 2/L$ and $|q_z|\sim 2/L$. These values are small compared with M_C^2 and can be neglected in the denominators. This reduces the r.h.s. of (3.24) to the form

$$< \bar{\chi}(x)\chi(x);\mathcal{E} > - < \bar{\chi}(0)\chi(0) >$$

$$=< \bar{\chi}(0)\chi(0) > \frac{M_C^2+\dfrac{g^2}{2G_1}}{M_C^2-\dfrac{g^2}{6G_1}}\frac{\kappa^2}{96\pi^2}\frac{1}{M^2}$$

$$\times\left[\int\limits_{-\infty}^{\infty}\frac{dk_z}{k_z}\sin\left(\frac{k_zL}{2}\right)\cos(k_zz)\right]^2$$

$$\times\iint\frac{d^2k_\perp}{2\pi^2}\frac{d^2q_\perp}{2\pi^2}\frac{(\mathbf{k}_\perp\cdot\mathbf{q}_\perp)}{(M_C^2+\mathbf{k}_\perp^2)(M_C^2+\mathbf{q}_\perp^2)}$$

$$\times e^{i\,(\mathbf{k}_\perp + \mathbf{q}_\perp)\cdot \mathbf{x}_\perp}. \qquad (3.27)$$

where $\mathbf{x}_\perp = (x, y)$. Taking into account that z is in the interval $-L/2 \le z \le L/2$ we simplify (3.26) as follows

$$< \bar{\chi}(x)\chi(x); \mathcal{E} > - < \bar{\chi}(0)\chi(0) >$$
$$= < \bar{\chi}(0)\chi(0) > \frac{M_C^2 + \dfrac{g^2}{2G_1}}{M_C^2 - \dfrac{g^2}{6G_1}} \frac{\kappa^2}{96\pi^2}\frac{1}{M^2}\iint \frac{d^2k_\perp}{2\pi}$$
$$\times \frac{d^2q_\perp}{2\pi}\frac{(\mathbf{k}_\perp\cdot\mathbf{q}_\perp)e^{i(\mathbf{k}_\perp + \mathbf{q}_\perp)\cdot \mathbf{x}_\perp}}{[M_C^2 + \mathbf{k}_\perp^2][M_C^2 + \mathbf{q}_\perp^2]}. \qquad (3.28)$$

We can represent the r.h.s. of (3.27) in the more convenient form

$$< \bar{\chi}(x)\chi(x); \mathcal{E} > - < \bar{\chi}(0)\chi(0) >$$
$$= - < \bar{\chi}(0)\chi(0) > \frac{M_C^2 + \dfrac{g^2}{2G_1}}{M_C^2 - \dfrac{g^2}{6G_1}} \frac{\kappa^2}{96\pi^2}\frac{1}{M^2}$$
$$\times \left[\bigtriangledown_{\mathbf{x}_\perp}\int \frac{d^2k_\perp}{2\pi}\frac{e^{i\,\mathbf{x}_\perp\cdot \mathbf{k}_\perp}}{M_C^2 + \mathbf{k}_\perp^2}\right]^2, \qquad (3.29)$$

where $\bigtriangledown_{\mathbf{x}_\perp}$ is the gradient with respect to $\mathbf{x}_\perp$.

Integrating over directions of the vector $\mathbf{k}_\perp$ and taking the gradient we get

$$< \bar{\chi}(x)\chi(x); \mathcal{E} > - < \bar{\chi}(0)\chi(0) >$$
$$= - < \bar{\chi}(0)\chi(0) > \frac{M_C^2 + \dfrac{g^2}{2G_1}}{M_C^2 - \dfrac{g^2}{6G_1}} \frac{\kappa^2}{96\pi^2}\frac{1}{M^2}$$
$$\times \left[\int_0^\infty \frac{dk k^2 J_1(kr)}{M_C^2 + k^2}\right]^2, \qquad (3.30)$$

where $J_1(uk)$ is a Bessel function and $r = |\mathbf{x}_\perp|$. The integral over k can be calculated explicitly and reads

$$\int_0^\infty \frac{dk k^2 J_1(kr)}{M_C^2 + k^2} = \frac{2M_C^2}{r}\int_0^\infty \frac{dk k J_0(kr)}{(M_C^2 + k^2)^2}$$
$$= M_C K_1(M_C r), \qquad (3.31)$$

where $K_1(M_C r)$ is a McDonald function.

Thus, the magnetic condensate averaged over quantum dual–vector field and string shape fluctuations reads

$$< \bar{\chi}(x)\chi(x); \mathcal{E} > = < \bar{\chi}(0)\chi(0) >$$
$$\times \left[1 - \frac{M_C^2 + \dfrac{g^2}{2G_1}}{M_C^2 - \dfrac{g^2}{6G_1}} \frac{\kappa^2}{96\pi^2}\frac{M_C^2}{M^2}K_1^2(M_C r)\right]. \qquad (3.32)$$

It may be seen that due to the constraint $M_C^2 > g^2/6G_1$ the magnetic monopole condensate at distances close to the string $r \to 0$ becomes suppressed. For $r \to 0$ the McDonald function $K_1(M_C r)$ behaves like $K_1(M_C r) \to 1/M_C r$. However, we have to emphasize that in such a model like the MNJL model [1,2] and a dual Higgs model [11] the region of distances close to the string is restricted by the constraint $r \ge 1/\Lambda_\perp$, where $\Lambda_\perp$ is the cut–off in plane perpendicular to the world–sheet of a dual Dirac string [2,5,11,13,14]. Due to Nambu [5] $1/\Lambda_\perp$ should be understood as a thickness of a string. Following [5,11] this cut–off $\Lambda_\perp$ should be identified with the mass of the σ–meson, , i.e. $\Lambda_\perp = M_\sigma = 2\,M$. As has been shown in [11] this choice makes next–to–leading order corrections in large M_σ expansion to the string tension logarithmically small compared with the leading order contribution. Thus, the McDonald function $K_1(M_C r)$ is restricted from above as $K_1(M_C r) \le 2M/M_C$. Since the value of the condensate can be either negative or zero, we can impose the constraint

$$1 - \frac{M_C^2 + \dfrac{g^2}{2G_1}}{M_C^2 - \dfrac{g^2}{6G_1}} \frac{\kappa^2}{24\pi^2} \ge 0, \qquad (3.33)$$

where we have neglected the contribution of the term of order $O(1/M^3)$. Using the relation $\kappa^2 = 2\,g^2/3$ we bring up (3.33) to the form

$$M_C^2 \ge \frac{g^2}{6G_1}\,\frac{1 + \dfrac{g^2}{12\pi^2}}{1 - \dfrac{g^2}{36\pi^2}}. \qquad (3.34)$$

This relation agrees with the inequality $M_C^2 > g^2/6G_1$ for any $g^2/36\pi^2 < 1$.

The inequalities $M_C^2 \ge g^2/6G_1$ and $g^2/36\pi^2 < 1$ can be rewritten as the inequality for the cut–off Λ and the monopole mass M. Using the relation $G_1 = G/4$, the expression for the mass of the dual–vector C_μ–field (1.2), the gap–equation (1.4) and the constraints (1.6) we obtain

$$\frac{39}{16}\left(\frac{\Lambda}{M}\right)^4 > 1 + \left(\frac{\Lambda}{M}\right)^2. \qquad (3.35)$$

The solution of this inequality gives only a trivial constraint $\Lambda > M$. This means that in the MNJL model with dual Dirac strings the suppression of the monopole condensate in the close vicinity of a dual Dirac string is always fulfilled.

At distances far from the string $r \to \infty$ the contribution of the string is exponentially suppressed as $e^{-2\,M_C r}$ due to the Meissner effect, and the magnetic monopole condensate tends to the magnitude of the order parameter, i.e. $< \bar{\chi}(0)\chi(0) >$. A similar influence of an electric flux tube, being an analogy to a dual Dirac string in CQED, on the magnitude of the magnetic monopole condensate has been observed within CQED [15].

4 Conclusion

We have investigated the magnetic monopole condensate as a functional of the shape of a dual Dirac string in the MNJL model assuming that the non–perturbative vacuum of the MNJL model is *Dual superconductor of type* II. The former has been realized through the constraint $M_\sigma \gg M_C$ [12]. Unlike a dual Higgs model with dual Dirac strings [11,12] the mass of a dual–vector field M_C is not proportional to the order parameter $<\bar{\chi}\chi>$ and does not vanish in the limit $<\bar{\chi}\chi>\to 0$. This is readily seen from the mass formula

$$M_\sigma^2\,(8\,M_C^2+3\,M_\sigma^2)=-56g^2<\bar{\chi}\chi>,$$

which can be derived from (1.2). Thus, in the MNJL model a dual–vector field does not need a Goldston boson as a longitudinal component. This distinguishes the transition to the non–perturbative superconducting phase in the NMJL and the dual Higgs model. Indeed, in the MNJL model this transition does not accompany the appearance of Goldston bosons. The former is rather natural, since the starting $U(1)$ magnetic symmetry in the MNJL model is global and unbroken in the non–perturbative superconducting phase. Recall that in the dual Higgs model the magnetic $U(1)$ symmetry is local and becomes spontaneously broken in the superconducting phase.

We have shown that the integration over quantum fluctuations of the dual–vector field C_μ around the shape of the Abrikosov flux line leads to a substantial contribution. In accordance with the prediction of CQED this contribution leads to the suppression of the magnetic monopole condensate at distances close to a dual Dirac string at the natural assumption that $\Lambda > M$ (see (3.35)).

At distances far from the string, where the influence of the string is exponentially suppressed due to the Meissner effect,the contribution of quantum dual–vector field fluctuations to the magnetic monopole condensate decreases. At infinitely large distances the magnitude of the magnetic monopole condensate tends to the magnitude of the order parameter, i.e., $<\bar{\chi}(0)\chi(0)>$.

The integration over string shape fluctuations can be performed analytically only for the fluctuations around the shape of the static straight string of length L. The contribution of the string shape fluctuations smoothes the suppression of the magnetic monopole condensate at distances close to the string and retains the exponential decrease at distances far from the string. The obtained results testify the equivalence of the MNJL model with dual Dirac strings to CQED.

Appendix
Regularization of the $\varphi(z)$–function

The function $\varphi(z)$ represented (3.25) is defined by a divergent expression. Therefore, it is requested to regularize it. For the regularization of $\varphi(z)$ we introduce an arbitrary infra–red parameter μ as follows

$$\varphi(z)\to\varphi(z)_{\rm R}=\frac{4}{3}\frac{g^2}{\pi}\frac{2}{L}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\times\frac{1}{\omega^2+\dfrac{4\pi^2n^2}{L^2}+\mu^2}\sin^2\left(\frac{2\pi n}{L}z\right).\quad\text{(A.1)}$$

The next step of the regularization is to apply the following integral representation:

$$\varphi(z)_{\rm R}=\lim_{\mu\to 0}\frac{4}{3}\frac{g^2}{\pi}\frac{2}{L}\times\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{dt}{2\pi}\,e^{-i2\pi nt/L}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega dp_z}{2\pi}\times\frac{e^{ip_zt}}{\omega^2+p_z^2+\mu^2}[1-\cos(2p_zz)].\quad\text{(A.2)}$$

It is easy to show that integrating over t and p_z we return to (A.1).

Then, it is convenient to decompose the integrals into two parts

$$\varphi(z)_{\rm R}=\varphi^{(1)}(z)_{\rm R}-\varphi^{(2)}(z)_{\rm R}\quad\text{(A.3)}$$

where we have denoted

$$\varphi^{(1)}(z)_{\rm R}=\frac{4}{3}\frac{g^2}{\pi}\frac{2}{L}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{dt}{2\pi}\,e^{-i2\pi nt/L}\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega dp_z}{2\pi}\frac{e^{ip_zt}}{\omega^2+p_z^2+\mu^2},\quad\text{(A.4)}$$

$$\begin{aligned}\varphi^{(2)}(z)_{\rm R}&=\frac{4}{3}\frac{g^2}{\pi}\frac{2}{L}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{dt}{2\pi}\,e^{-i2\pi nt/L}\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega dp_z}{2\pi}\frac{e^{ip_zt}}{\omega^2+p_z^2+\mu^2}\cos(2p_zz)\\&=\frac{2}{3}\frac{g^2}{\pi}\frac{2}{L}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}\frac{dt}{2\pi}\,e^{-i2\pi nt/L}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega dp_z}{2\pi}\times\frac{e^{ip_z(t+2z)}+e^{ip_z(t-2z)}}{\omega^2+p_z^2+\mu^2}.\end{aligned}\quad\text{(A.5)}$$

Now let us perform a summation over index n which gives

$$\varphi^{(1)}(z)_{\rm R}=\frac{4}{3}\frac{g^2}{\pi}\frac{2}{L}\int_{-\infty}^{\infty}\frac{dt}{4\pi i}\frac{e^{i\pi t/L}}{\sin(\pi t/L)}\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{d\omega dp_z}{2\pi}\frac{e^{ip_zt}}{\omega^2+p_z^2+\mu^2},\quad\text{(A.6)}$$

$$\varphi^{(2)}(z)_{\rm R} = \frac{2}{3}\frac{g^2}{\pi}\frac{2}{L}\int\limits_{-\infty}^{\infty}\frac{dt}{4\pi i}\,\frac{e^{i\pi t/L}}{\sin(\pi t/L)}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\frac{d\omega dp_z}{2\pi}\times\frac{e^{ip_z(t+2z)}+e^{ip_z(t-2z)}}{\omega^2+p_z^2+\mu^2}. \qquad (A.7)$$

It is convenient to proceed to polar coordinates in the plane (ω, p_z) and perform the integration over the azimuthal angle:

$$\varphi^{(1)}(z)_{\rm R} = \frac{4}{3}\frac{g^2}{\pi}\frac{2}{L}\int\limits_{-\infty}^{\infty}\frac{dt}{4\pi i}\times\frac{e^{i\pi t/L}}{\sin(\pi t/L)}\int\limits_{0}^{\infty}\frac{dp\,p\,J_0(pt)}{p^2+\mu^2}, \qquad (A.8)$$

$$\varphi^{(2)}(z)_{\rm R} = \frac{2}{3}\frac{g^2}{\pi}\frac{2}{L}\int\limits_{-\infty}^{\infty}\frac{dt}{4\pi i}\,\frac{e^{i\pi t/L}}{\sin(\pi t/L)}\int\limits_{0}^{\infty}\frac{dp\,p}{p^2+\mu^2}\times[J_0(p(t+2z))+J_0(p(t-2z))], \qquad (A.9)$$

where $J_0(x)$ is a Bessel function. Since the Bessel functions in the integrand of (A.8) and (A.9) are even under the transformation $t \to -t$, the integrals become

$$\varphi^{(1)}(z)_{\rm R} = \frac{4}{3}\frac{g^2}{\pi}\frac{2}{L}\int\limits_{-\infty}^{\infty}\frac{dt}{4\pi}\int\limits_{0}^{\infty}\frac{dp\,p\,J_0(pt)}{p^2+\mu^2}, \qquad (A.10)$$

$$\varphi^{(2)}(z)_{\rm R} = \frac{2}{3}\frac{g^2}{\pi}\frac{2}{L}\int\limits_{-\infty}^{\infty}\frac{dt}{4\pi}\int\limits_{0}^{\infty}\frac{dp\,p}{p^2+\mu^2}\times[J_0(p(t+2z))+J_0(p(t-2z))]. \qquad (A.11)$$

The dependence of z can be removed by the shifts $t+2z \to t$ and $t-2z \to t$:

$$\varphi^{(1)}(z)_{\rm R} = \varphi^{(2)}(z)_{\rm R} = \frac{4}{3}\frac{g^2}{\pi}\frac{2}{L}\times\int\limits_{-\infty}^{\infty}\frac{dt}{4\pi}\int\limits_{0}^{\infty}\frac{dp\,p\,J_0(pt)}{p^2+\mu^2}, \qquad (A.12)$$

As the integral over t equals to

$$\int\limits_{-\infty}^{\infty}dt\,p\,J_0(pt) = 2, \qquad (A.13)$$

the functions $\varphi^{(1)}(z)_{\rm R}$ and $\varphi^{(2)}(z)_{\rm R}$ are defined by the integral over p:

$$\varphi^{(1)}(z)_{\rm R} = \varphi^{(2)}(z)_{\rm R} = \frac{2}{3}\frac{g^2}{\pi^2}\frac{2}{L}\times\int\limits_{0}^{\infty}\frac{dp}{p^2+\mu^2} = \frac{g^2}{3\pi}\frac{2}{L\mu}. \qquad (A.14)$$

Substituting (A.13) in (A.3) we get

$$\varphi(z)_{\rm R} = 0. \qquad (A.15)$$

Thus, the regularized version of the $\varphi(z)$–function vanishes for all $z \in [-L/2, L/2]$.

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